Reaction Coordinate Master Equation for Transport Problems Beyond Born-Markov

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May 16, 2023

Outline

Question 1

How would a **quantum** system evolve in contact with a thermal environment?

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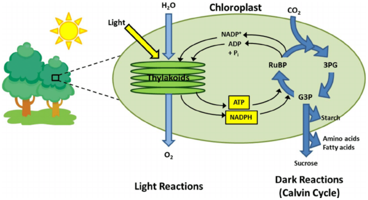
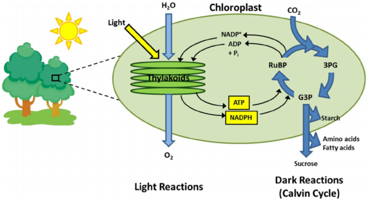
How would a **quantum** system evolve in contact with a thermal environment? $\leftarrow$ Why is this interesting?

Quantum systems in contact with a thermal bath (in nature)



- Photosynthesis is at room temperature

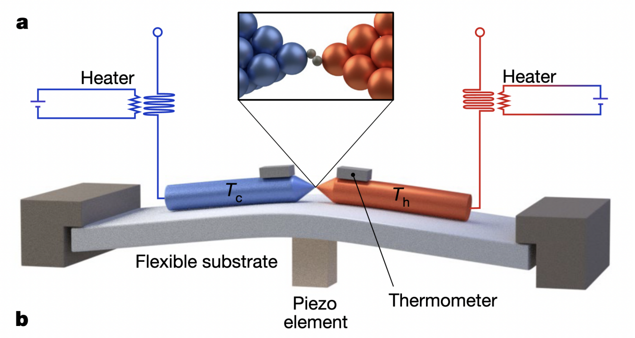
Quantum systems in contact with a thermal bath (in nature)



- Photosynthesis is at room temperature
- Quantum effects in photosynthesis

Quantum biology...

Quantum systems in contact with a thermal bath (...in the lab)



- Atomic junction experiments[1]
- Quantum system as a conductor

[1] Ofir Shein Lumbroso, Lena Simine, Abraham Nitzan, Dvira Segal, and Oren Tal, Nature 2018

How would a **quantum** system evolve in contact with a thermal environment?

- **Quantization?** Feynman: thermal environment $\rightarrow$ infinitely many harmonic oscillators.

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But...

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But... Dirac:

- ...laws necessary for the ...large part of physics and the whole of chemistry are thus completely known,
- ... the difficulty is only that the **exact** application of these laws leads to equations much too complicated to be soluble...

i.e., $|\psi_{S+E}(t)\rangle$ is huge **but** we do not care about the environment part. One solution is to use a dissipative master equation.

Lindblad equation: Top-Down (short time expansion of the Kraus operator)

The reduced system density matrix satisfies

$$\langle i|\rho|i\rangle \geq 0 \tag{1}$$
$$\mathrm{Tr}\{\rho\} = 1 \tag{2}$$

Therefore, we'd like to find a quantum map that preserves these properties,

$$\rho \to \rho', \quad \text{via} \quad \dot{\rho} = \mathcal{L}\rho \tag{3}$$

[2] Breuer and Petruccione or Lidar are good references

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which Lindblad proved to take the general GKLS form[2]

$$\dot{\rho} = \underbrace{-i[\hat{H}, \rho]}_{\text{unitary}} + \underbrace{\sum_k \Gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2}\left\{ L_k^\dagger L_k, \rho \right\} \right)}_{\text{dissipator}} \equiv \mathcal{L}\rho. \tag{4}$$

[2] Breuer and Petruccione or Lidar are good references

Master Equations: Bottom-up (perturbative)

Schrodinger's Equation

$\downarrow$

Von Neumann

$\downarrow$ Born-Markov, Tr_B

Redfield QME

$\downarrow$ Secular

Lindblad QME

Lindblad is Secular Redfield

The bottom up derivation of Lindblad equation is Redfield (Born-Markov) + Rotating Wave (Secular) Approximation.
We will stop at Redfield to go beyond secular, but note that Redfield is notoriously non-CPTP.

Born-Markov Redfield: Primer

The full Hamiltonian takes the form

$$\hat{H} = \underbrace{\hat{H}_S + \hat{H}_B}_{\hat{H}_0} + \hat{V}, \tag{5}$$

with

$$\hat{H}_B = \sum_j \omega_j \hat{b}_j^\dagger \hat{b}_j. \tag{6}$$

The system-bath interaction Hamiltonian is bilinear

$$\hat{V} = \hat{S} \otimes \hat{B}; \quad \hat{B} = \sum_j g_j(\hat{b}_j^\dagger + \hat{b}_j). \tag{7}$$

g_j describes the system-bath coupling energy between mode j in the bath and the system.

Born-Markov Redfield: Derivation Sketch

- Starting from von Neumann equation in the interaction picture,

$$\dot{\rho}_I(t) = -i[\hat{V}_I, \rho_I(t)] \tag{8}$$

Born-Markov Redfield: Derivation Sketch

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$$\dot{\rho}_I(t) = -i[\hat{V}_I, \rho_I(t)] \tag{8}$$

- Make the **Born** approximation, i.e., $\rho \approx \rho_S \otimes \rho_B$ and "$\hat{V} \ll \hat{H}_0$"

$$\underbrace{\frac{\partial \rho_I}{\partial t}(t) = -i[\hat{V}_I(t), \rho_I(t_0)] - \int_{t_0}^{t} d\tau [\hat{V}_I(t), [\hat{V}_I(\tau), \rho_I(\tau)]]}_{\text{Partial trace} \Rightarrow \frac{\partial \rho_{S,I}}{\partial t}(t) = \underbrace{-i\,\mathrm{Tr}_B\{[\hat{V}_I(t), \rho_I(t_0)]\}}_{\text{0 for a harmonic bath}} - \mathrm{Tr}_B\{\int_{t_0}^{t} d\tau [\hat{V}_I(t), [\hat{V}_I(\tau), \rho_I(\tau)]]\}} \quad ($$

Born-Markov Redfield: Derivation Sketch

- **Markov I** (also stationary bath)

$$\frac{\partial \rho_{S,I}(t)}{\partial t} = -\,\mathrm{Tr}_B\{\int_{t_0}^{t} d\tau [\hat{V}_I(t), [\hat{V}_I(\tau), \rho_{S,I}(t) \otimes \rho_B]]\}, \qquad (10)$$

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- **Markov II**:

$$\frac{\partial \rho_{S,I}(t)}{\partial t} = -\mathrm{Tr}_B\{\int_{0}^{\infty} d\tau [\hat{V}_I(t), [\hat{V}_I(t-\tau), \rho_{S,I}(t) \otimes \rho_B]]\}. \tag{11}$$

Markov: memoryless, "What happens next depends only on the state of affairs now.". For example, drunkard's walk **is** Markov but Bus waiting is **not** Markov.

Born-Markov Redfield: Derivation Sketch

Rotate back to the Schrödinger picture and do algebra.

$$\frac{\partial \rho_s}{\partial t} = -\frac{i}{\hbar}[\hat{H}_s, \rho_s] - \int_0^\infty \Big\{ [\hat{S}, e^{-i\hat{H}_s\tau}\hat{S}e^{i\hat{H}_s\tau}\rho_s(t)]\langle \hat{B}_I(t-\tau)\hat{B}_I(t)\rangle - [\hat{S}, \rho_s(t)e^{-i\hat{H}_s\tau}\hat{S}e^{i\hat{H}_s\tau}]\langle \hat{B}_I(t)\hat{B}_I(t-\tau)\rangle \Big\} d\tau \tag{12}$$

we'll eventually need to Laplace transform the bath correlation function

$$\underbrace{R_{ij,kl}(\omega)}_{\text{for Redfield Liouvillian}} = S_{ij}S_{kl}\int_0^\infty d\tau e^{i\omega\tau} \underbrace{\langle \hat{B}_I(\tau)\hat{B}_I(0)\rangle}_{\sum_j \lambda_j^2[e^{i\omega_j t}\langle \hat{n}(\omega_j)\rangle + e^{-i\omega_j t}\langle \hat{n}(\omega_j)+1\rangle]} \tag{13}$$

The Sokhotski–Plemelj theorem says

$$\lim_{\epsilon\to 0^+}\frac{1}{x \pm i\epsilon} = \mp i\pi\delta(x) + \mathcal{P}(\frac{1}{x}), \tag{14}$$

Spectral Density

The real part of the Laplace transform $\Gamma(\omega)$ matters. (the imaginary part is a negligible Lamb shift). Notice that we'll find a delta term

$$\Gamma_\alpha(\omega) = \begin{cases} \pi J_\alpha(\omega) n_\alpha(|\omega|) & \omega < 0, \\ \pi J_\alpha(\omega)[(n_\alpha(\omega) + 1] & \omega > 0, \\ \pi C_\alpha & \omega = 0, \end{cases} \tag{15}$$

$$J(\omega) = \sum_k \lambda_k^2 \delta(\omega - \omega_k), \tag{16}$$

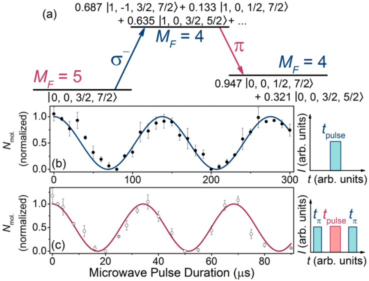
all we need to know about the environment is encoded in the spectral density $J(\omega)$.

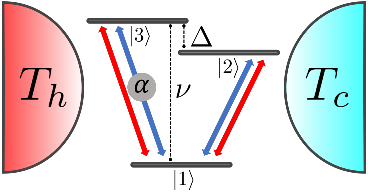
Some remarks on Born-Markov Redfield QME

- Redfield QME is used all the time, especially for complex problems where microscopic details are important, e.g., in quantum thermodynamics, quantum biology, etc.
- Assumptions:
  - Born (Weak coupling) $\rightarrow$ second order in the system bath coupling parameter
  - Markov (Memoryless)
- But, unlike Lindblad, there is no secular approximation

Secular approximation

Fails for systems with near-degenerate levels, such as those used for (1) adiabatic quantum computing, (2) coherent population trapping and electromagnetically induced transparency, where coherences are prominent[3,4]. This is because secular approximation decouples population and coherence dynamics.





[3] FI, Nicholas Anto-Sztrikacs, and Dvira Segal, NJP 2022

[4] FI, Nicholas Anto-Sztrikacs, and Dvira Segal, arxiv:2301.06135, 2023

Question 2:

How to go beyond Born-Markov?

- Fully Numerical:
  - Multiconfiguration time-dependent Hartree (MCTDH)
  - Hierarchical equations of motion (HEOM) (Tanimura)
  - Density matrix renormalization group (DMRG)
  - Numerical path integral (Segal, Millis, and Reichman, 2010 PRB) $\leftarrow$ in the journal club suggestion list
  - **Chain-mapping methods**, particularly TEDOPA (Chin and Plenio)
  - Tensor network methods (Cao, Huelga, Plenio)
  - Quantum monte-carlo

  i.e., solve cleverly the $S + B$ full dynamics.

Question 2:

- Inexact analytical:
  - Non-interacting blip approximation (NIBA) (Segal)
  - Polaron-transformation (Cao, Segal, Silbey, Cheng, etc)
  - Green's function techniques

  each is applicable in very particular circumstances.

Question 2:

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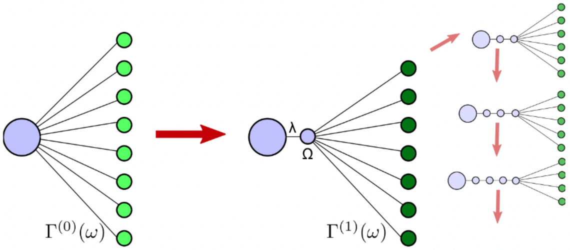
  - Green's function techniques

  each is applicable in very particular circumstances.
- the reaction-coordinate quantum master equation method is in between: a *semi*-analytical method.

Reaction Coordinate Mapping: Primer (Chain Mapping)

Recall that the quantum system is coupled to many harmonic oscillators...

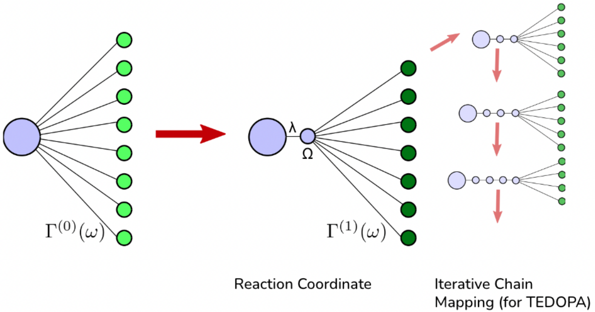


Reaction Coordinate

Iterative Chain Mapping (for TEDOPA)

Reaction Coordinate Mapping: Primer (Chain Mapping)

Recall that the quantum system is coupled to many harmonic oscillators...



- A couple words on TEDOPA... (a) numerically exact mapping through orthogonal polynomials, (b) infinitely long chain $\rightarrow$ truncated chain (bounded by Lieb-Robinson technique) + truncated harmonic manifold, (c) evolved with DMRG or TEBD, essentially evolving the whole chain, must Trotterize.

Reaction Coordinate Mapping: Details

$$\hat{H} = \hat{H}_s + \sum_k \nu_k \left(\hat{c}_k^\dagger + \hat{S}\frac{f_k}{\nu_k} \right) \left(\hat{c}_k + \hat{S}\frac{f_k}{\nu_k} \right) \tag{17}$$

$$\downarrow$$

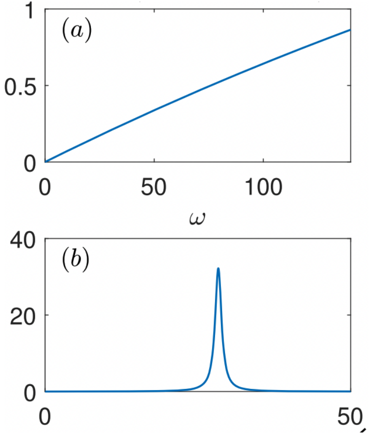
$$\begin{aligned} \hat{H} \;&=\; \hat{H}_s + \Omega \left(\hat{a}^\dagger + \frac{\lambda}{\Omega}\hat{S} \right) \left(\hat{a} + \frac{\lambda}{\Omega}\hat{S} \right) \\ &\quad + \sum_k \omega_k \left(\hat{b}_k^\dagger + (\hat{a} + \hat{a}^\dagger)\frac{f_k}{\omega_k} \right) \left(\hat{b}_k + (\hat{a} + \hat{a}^\dagger)\frac{f_k}{\omega_k} \right) \end{aligned} \tag{18}$$

where $\lambda(\hat{a}^\dagger + \hat{a}) = \sum_k f_k(\hat{c}^\dagger + \hat{c})$. Note that

- The system Hamiltonian (Red) expands
- The coupling is redrawn, from initial system$\rightarrow$bath to extracted mode$\rightarrow$residual bath

Reaction Coordinate Mapping: Details

Also, $J(\omega) \to J_{RC}(\omega)$ (quite technical, see[5]). A fair simplification is from a Brownian (peaked) J about Ω (b) $\to$ an Ohmic (linear) J_{RC} (a)



[5] Nicholas Anto-Sztrikacs and Dvira Segal, 2021 NJP

Some remarks on the RCQME

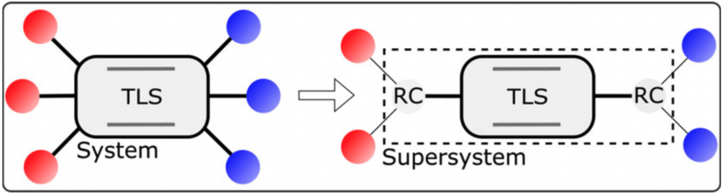
- After the mapping, we perform BMR-QME, as the residual system-bath coupling parameter is small.
- A truncation is performed on the reaction mode, so that the extended system Hamiltonian is finite.
  - Hence, RCQME is not intended for high-temperature dynamics.
  - The extended Hamiltonian scales as $(\#_{\text{system levels}})(\#_{\text{extracted manifold}})^{\#_{\text{extracted bath}}}$. Numerical complexity $\propto$ power 4th of extended Hamiltonian dimension to construct Redfield tensor.
- A partial trace over the reaction modes is then taken to revert back to the (original) system basis.
- Can use existing toolbox developed for BMR-QME or Lindblad QME.

Applications of the RCQME (from the Segal group)

- Mostly numerical:
  - Non-equilibrium spin-boson at strong coupling[6]
  - **Quantum absorption refrigerator at strong coupling**[7]
  - Markovian dynamics[8]
- Analytical:
  - **Transport beyond second order**[9]
  - Generalized effective hamiltonian theory[10]

[6] Nicholas Anto-Sztrikacs and Dvira Segal, 2021 NJP
[7] FI*, NAS*, and DS, 2022 PRE
[8] NAS and DS, 2021 PRA
[9] NAS, FI, and DS, 2022 JCP
[10] NAS, Ahsan Nazir, and DS, 2023 PRX Quantum

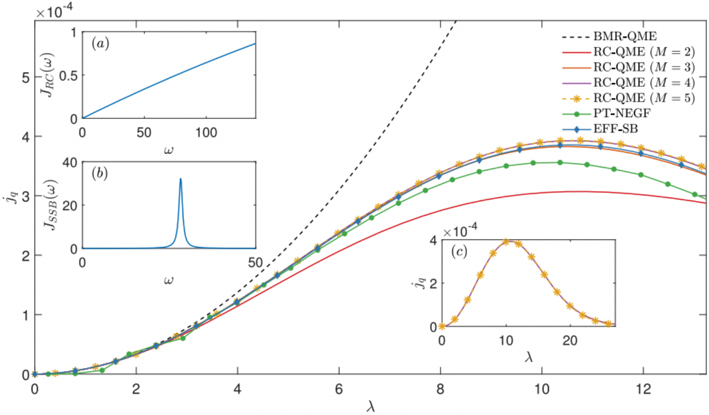
Non-equilibrium spin-boson at strong coupling[11]



We'd like to know $j_{q,i} = \mathrm{Tr}\left\{ D_i(\rho_{ES})\hat{H}_{ES} \right\}$ (i.e., how conductive the qubit is) at steady state

[11] Nicholas Anto-Sztrikacs and Dvira Segal, 2021 NJP

Non-equilibrium spin-boson at strong coupling[12]



- RC-QME captures a signature of strong-coupling transport, turnover.
- RC-QME agrees with numerically intensive methods, PT-NEGF.

[12] Nicholas Anto-Sztrikacs and Dvira Segal, 2021 NJP

Signature of strong coupling: energy renormalization

Energy renormalization causes turnover behaviour. At low temperature...

- Squeeze slightly ⇒ low cost to excite effective qubit ⇒ higher current
- Squeeze too much ⇒ each photon carries little energy ⇒ lower current

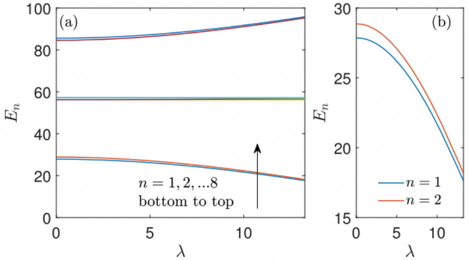
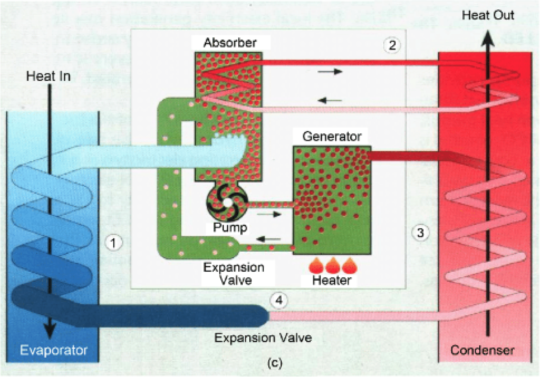


Figure 2. (a) Eigenenergies of $H_{\mathrm{ES}}^{M=2}$ with $\Delta = 1$, $\varepsilon = 0$, $\Omega = 28\Delta$ [65]. (b) Focus on the lowest two eigenvalues, which form an effective spin Hamiltonian.

Quantum Absorption Refrigerator at strong coupling[13]

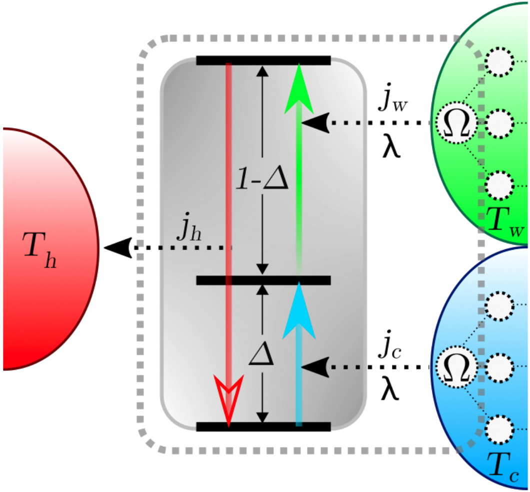
an Absorption Refrigerator takes in heat from T_c and dumps it to T_h using work from T_w ($T_w > T_h > T_c$).



[13] FI*, NAS*, and DS, 2022 PRE

Quantum Absorption Refrigerator at strong coupling[14]

This refrigerator is therefore quantummable.



[14] FI*, NAS*, and DS, 2022 PRE

Quantum Absorption Refrigerator at strong coupling[15]

In the tight-coupling limit (i.e., one quanta in one quanta out) one can prove

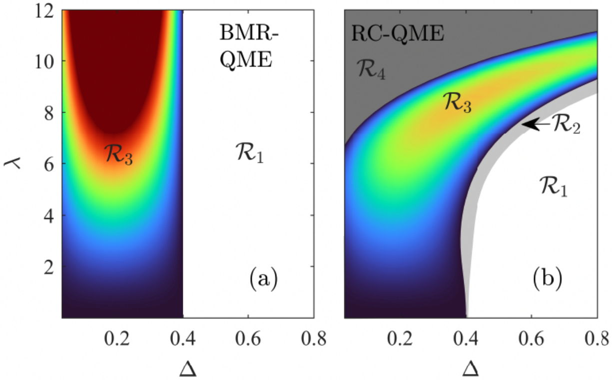
$$\frac{\epsilon_2 - \epsilon_1}{\epsilon_3 - \epsilon_1} \leq \frac{\beta_h - \beta_w}{\beta_c - \beta_w} \Leftrightarrow \text{cooling} \tag{19}$$

also, from BMR-QME, higher λ leads to better cooling.

[15] FI*, NAS*, and DS, 2022 PRE

Quantum Absorption Refrigerator at strong coupling[16]

With strong coupling,

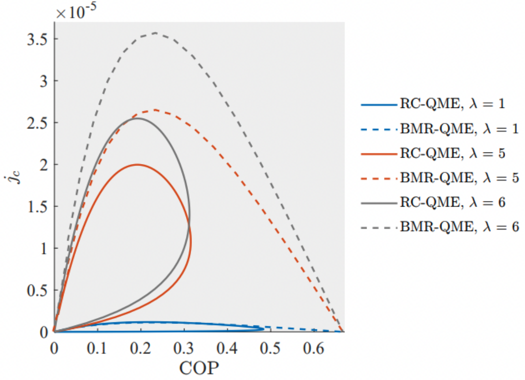


- Reshaping of cooling region, $\mathcal{R}_3$ (Renormalization)
- Emergence of new transport pathways , $\mathcal{R}_2$ (Bath-bath pathway)

[16] FI*, NAS*, and DS, 2022 PRE

Quantum Absorption Refrigerator at strong coupling[17]

And therefore we'll never hit Carnot's efficiency,



[17] FI*, NAS*, and DS, 2022 PRE

Quantum transport beyond second order

Recall in the derivation of BMR-QME, we cut the Dyson series to second order. Some nontrivial effects can arise **even** at weak coupling, if we had kept on going.

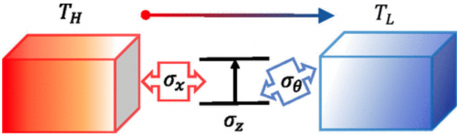
One example is the $\sigma_x - \sigma_z$ type transport reported in Ref.[18]

[18] Jianshu Cao et al., JCP 2020

Quantum transport beyond second order

Consider the generalized non-equilibrium spin-boson (NESB) model,

$$\begin{aligned}\hat{H}_{SB} &= \frac{\Delta}{2}\hat{\sigma}_z + \hat{\sigma}_x \sum_k f_{k,L}(\hat{c}^\dagger_{k,L} + \hat{c}_{k,L}) \\ &+ \underbrace{\hat{\sigma}_\theta}_{\hat{\sigma}_z \cos(\theta) + \hat{\sigma}_x \sin(\theta)} \sum_k f_{k,R}(\hat{c}^\dagger_{k,R} + \hat{c}_{k,R}) \\ &+ \sum_{k,\alpha\in\{R,L\}} \nu_{k,\alpha}\hat{c}^\dagger_{k,\alpha}\hat{c}_{k,\alpha}.\end{aligned} \tag{20}$$



Quantum transport beyond second order

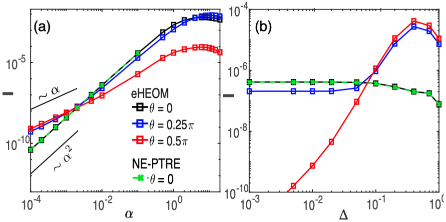
It could be shown that the heat current at steady state takes

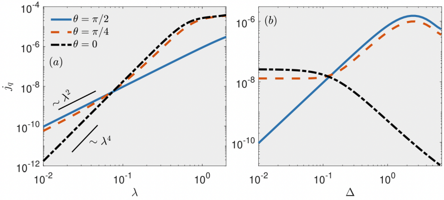
$$\begin{aligned} j_q &\equiv -\langle \dot{\hat{H}}_{B,L} \rangle = -i\langle [\hat{H}, \hat{H}_{B,L}] \rangle, \\ &= -i\lambda_L^2 \langle [\hat{H}_S, \hat{V}_L] \rangle - i\lambda_L^2 \lambda_R^2 \langle [\hat{V}_R, \hat{V}_L] \rangle. \end{aligned} \tag{21}$$

the first term is captured by second-order BMR-QME, but the second term is not. The latter is an **interbath** transport pathway, which also appeared as leakage for QAR at strong coupling.

Quantum transport beyond second order

Numerically intensive HEOM and NE-PTRE captures $j_q \propto \lambda^4$ current[19], but RC-QME captures them just as well[20].





[19] Jianshu Cao et al., JCP 2020

[20] NAS, FI, and DS, JCP 2022

Hints of analyticity with the RC-QME

The Polaron transform (the PT in NE-PTRE) is used to treat strong-coupling effects in very particular cases, modifying $\hat{V}$ in return for dressing $\hat{H}_s$. Performing PT post reaction-coordinate mapping reveals interesting analytical results.

The RC-mapped Hamiltonian for Eq. (20) is

$$
\begin{aligned}
\hat{H}_{SB-RC} &= \frac{\Delta}{2}\hat{\sigma}_z + \Omega_L \hat{a}_L^\dagger \hat{a}_L + \Omega_R \hat{a}_R^\dagger \hat{a}_R + \lambda_L \hat{\sigma}_x (\hat{a}_L^\dagger + \hat{a}_L) \\
&+ \lambda_R \hat{\sigma}_\theta (\hat{a}_R^\dagger + \hat{a}_R) + \sum_{k,\alpha} \frac{g_{k,\alpha}^2}{\omega_{k,\alpha}} (\hat{a}_\alpha^\dagger + \hat{a}_\alpha)^2 \\
&+ \sum_\alpha (\hat{a}_\alpha^\dagger + \hat{a}_\alpha) \sum_k g_{k,\alpha} (\hat{b}_{k,\alpha}^\dagger + \hat{b}_{k,\alpha}) + \sum_{k,\alpha} \omega_{k,\alpha} \hat{b}_{k,\alpha}^\dagger \hat{b}_{k,\alpha}.
\end{aligned} \tag{22}
$$

PT-RCQME

Transform that Hamiltonian, $\hat{\tilde{H}}_{SB-RC} = \hat{U}_P \hat{H}_{SB-RC} \hat{U}_P^\dagger$, to the left reservoir with $\hat{U}_P^L = e^{\frac{\lambda_L}{\Omega_L}\hat{\sigma}_x(\hat{a}_L^\dagger - \hat{a}_L)}$. This results to

At low temperatures, $\Delta \to \Delta e^{-\frac{\lambda^2}{2\Omega^2}}$

$$
\begin{aligned}
\hat{\tilde{H}}_{SB-RC} = & \boxed{\frac{\Delta}{4}\left[(\hat{\sigma}_z - i\hat{\sigma}_y)\, e^{\frac{\lambda_L}{\Omega_L}(\hat{a}_L^\dagger - \hat{a}_L)} + (\hat{\sigma}z + i\hat{\sigma}_y)\, e^{-\frac{\lambda_L}{\Omega_L}(\hat{a}_L^\dagger - \hat{a}_L)}\right]} + \Omega_R \hat{a}_R^\dagger \hat{a}_R + \Omega_L \hat{a}_L^\dagger \hat{a}_L \\
& + \lambda_R \sin\theta(\hat{a}_R^\dagger + \hat{a}_R)\hat{\sigma}_x + \lambda_R \cos\theta(\hat{a}_R^\dagger + \hat{a}_R)\frac{1}{2}\left[(\hat{\sigma}_z - i\hat{\sigma}_y)\, e^{\frac{\lambda_L}{\Omega_L}(\hat{a}_L^\dagger - \hat{a}_L)} + (\hat{\sigma}z + i\hat{\sigma}_y)\, e^{-\frac{\lambda_L}{\Omega_L}(\hat{a}_L^\dagger - \hat{a}_L)}\right] \\
& - \frac{2\lambda_L}{\Omega_L}\hat{\sigma}_x \sum_k g_{k,L}(\hat{b}_{k,L}^\dagger + \hat{b}_{k,L}) + (\hat{a}_L^\dagger + \hat{a}_L - \frac{2\lambda_L}{\Omega_L}\hat{\sigma}_x)^2 \sum_k \frac{g_{k,L}^2}{\omega_{k,L}} + (\hat{a}_R^\dagger + \hat{a}_R)^2 \sum_k \frac{g_{k,R}^2}{\omega_{k,R}} \\
& + \sum_\alpha (\hat{a}_\alpha^\dagger + \hat{a}_\alpha) \sum_k g_{k,\alpha}(\hat{b}_{k,\alpha}^\dagger + \hat{b}_{k,\alpha}) + \sum_{k,\alpha} \omega_{k,\alpha} \hat{b}_{k,\alpha}^\dagger \hat{b}_{k,\alpha}.
\end{aligned}
$$

PT-RCQME

Set $\theta = \pi/2$ and perform an additional polaron transform on the right reservoir. At low temperatures and to lowest order in λ,

$$
\begin{aligned}
\hat{H}_{SB-RC}^{\sigma_x-\sigma_x} &= \frac{\Delta}{2}\hat{\sigma}_z - \frac{\Delta}{2} i\hat{\sigma}_y \sum_\alpha \frac{\lambda_\alpha}{\Omega_\alpha}(\hat{a}_\alpha^\dagger - \hat{a}_\alpha) + \Omega_R \hat{a}_R^\dagger \hat{a}_R + \Omega_L \hat{a}_L^\dagger \hat{a}_L \\
&+ \boxed{\sum_\alpha (\hat{a}^\dagger_\alpha + \hat{a}_\alpha)\left[\hat{\sigma}_x\left(\frac{-2\lambda_\alpha}{\Omega_\alpha}\right)\sum_k \frac{g_{k,\alpha}^2}{\omega_{k,\alpha}} + \sum_k g_{k,\alpha}(\hat{b}^\dagger_{k,\alpha}+\hat{b}_{k,\alpha})\right]} \\
&- \sum_\alpha \frac{2\lambda_\alpha}{\Omega_\alpha}\hat{\sigma}_x \sum_k g_{k,\alpha}(\hat{b}^\dagger_{k,\alpha}+\hat{b}_k) + \sum_{k,\alpha}\omega_{k,\alpha}\hat{b}^\dagger_{k,\alpha}\hat{b}_{k,\alpha} \\
&+ \sum_\alpha(\hat{a}^\dagger_\alpha+\hat{a}_\alpha)^2 \sum_k \frac{g^2_{k,\alpha}}{\omega_{k,\alpha}}.
\end{aligned}
$$

One bath excites RC $\rightarrow$ RC excites system via σ_x $\rightarrow$ the other bath.

PT-RCQME

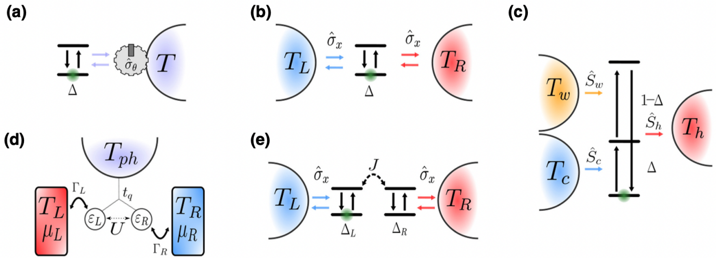
Set $\theta = 0$,

$$\begin{aligned}
\hat{H}^{\sigma_x-\sigma_z}_{SB-RC} &= \frac{\Delta}{2}\hat{\sigma}_z - \frac{i\lambda_L\Delta}{2\Omega_L}\hat{\sigma}_y(\hat{a}^\dagger_L - \hat{a}_L) + \sum_{k,\alpha}\omega_{k,\alpha}\hat{b}^\dagger_{k,\alpha}\hat{b}_{k,\alpha} \\
&+\Omega_R\hat{a}^\dagger_R\hat{a}_R + \Omega_L\hat{a}^\dagger_L\hat{a}_L + \lambda_R\hat{\sigma}_z(\hat{a}^\dagger_R + \hat{a}_R) \\
&-\boxed{i\frac{\lambda_R\lambda_L}{\Omega_L}(\hat{a}^\dagger_R + \hat{a}_R)\hat{\sigma}_y(\hat{a}^\dagger_L - \hat{a}_L)} \\
&-\frac{2\lambda_L}{\Omega_L}\hat{\sigma}_x\sum_k g_{k,L}(\hat{b}^\dagger_{k,L} + \hat{b}_{k,L}) + (\hat{a}^\dagger_L + \hat{a}_L - \frac{2\lambda_L}{\Omega_L}\hat{\sigma}_x)^2\sum_k\frac{g^2_{k,L}}{\omega_{k,L}} \\
&+(\hat{a}^\dagger_R + \hat{a}_R)^2\sum_k\frac{g^2_{k,R}}{\omega_{k,R}} + \sum_\alpha(\hat{a}^\dagger_\alpha + \hat{a}_\alpha)\sum_k g_{k,\alpha}(\hat{b}^\dagger_{k,\alpha} + \hat{b}_{k,\alpha})
\end{aligned}$$

Emergence of an unusual bath-bath transport pathway which scales differently as the bath-system-bath pathway.

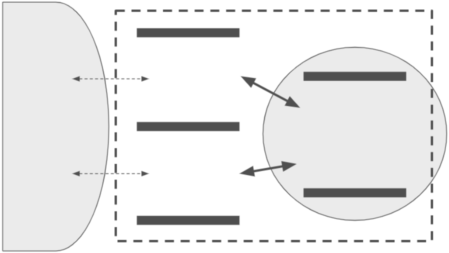
PT-RCQME was generalized by NAS, where strong coupling effects can be encoded at the Hamiltonian level simple enough to do analytical work[21]

$$\hat{H}_s^{\text{eff}}(\lambda) = \langle 0| e^{(\lambda/\Omega)(\hat{a}^\dagger - \hat{a})\hat{S}} \hat{H}_s e^{-(\lambda/\Omega)(\hat{a}^\dagger - \hat{a})\hat{S}} |0\rangle.$$



[21] NAS, Ahsan Nazir, and DS, PRX Quantum 2023

Markovian dynamics with RC-QME[22]



[22] NAS and DS, 2021 PRA

Conclusion

Reaction-coordinate master equation is an semi-analytical method to study quantum dynamics beyond Born-Markov. Signatures of strong coupling can explain complex models, for example quantum absorption refrigerators.

Acknowledgements

- Dvira Segal
- Nicholas Anto-Sztrikacs